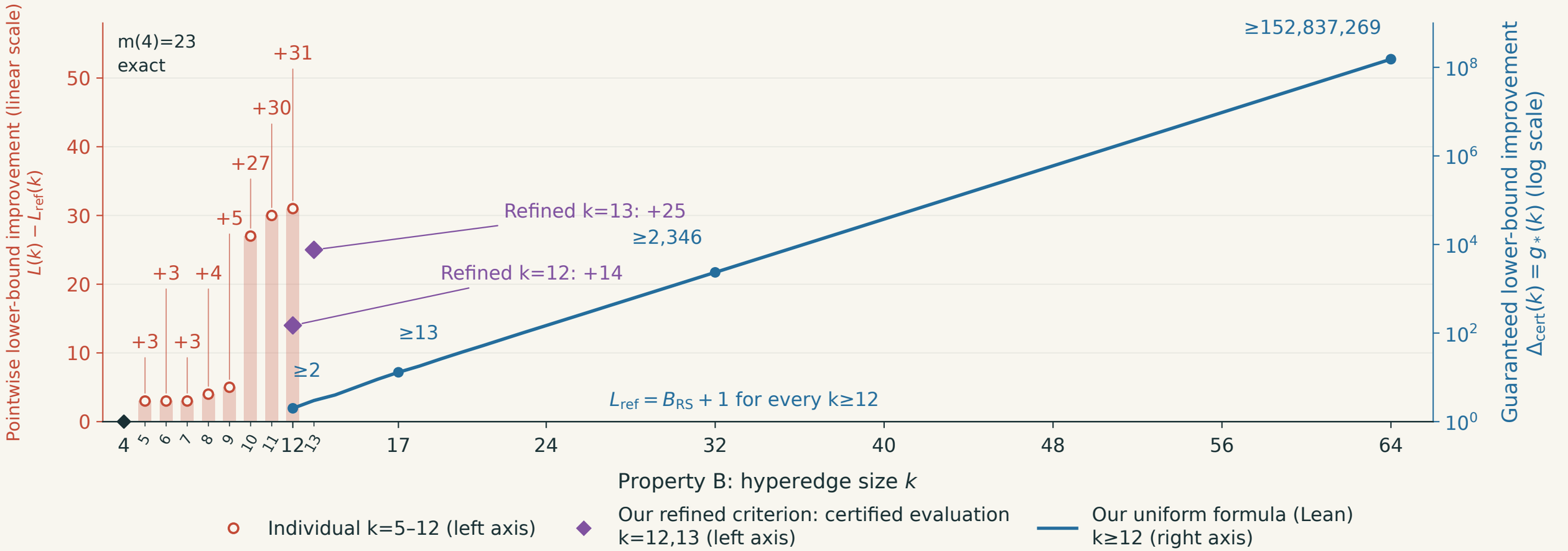
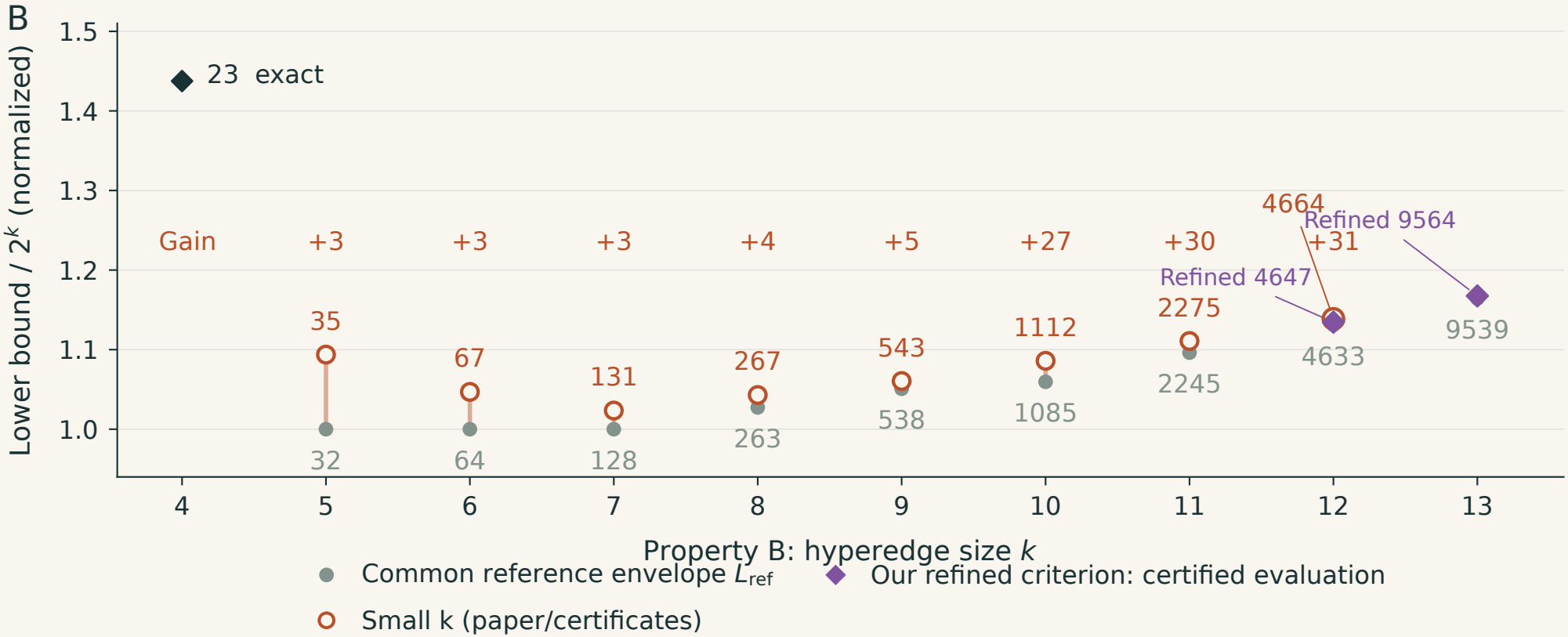
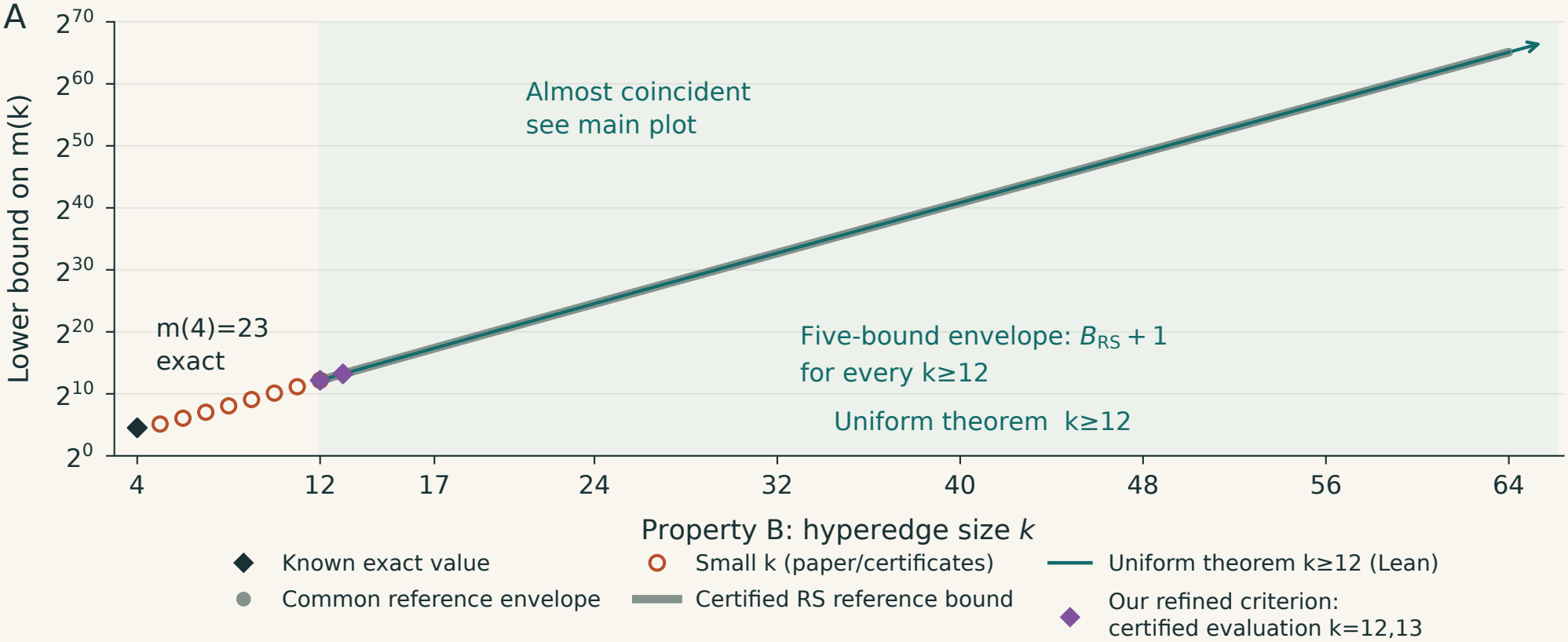


$\Delta_{\mathrm{cert}}(k) = g_*(k) \le \Delta(k)$; the right curve shows a certified lower bound on the gain.
Same refined criterion: direct evaluation (purple); uniform guarantee (blue).
Same reference envelope; independent y scales. Compare gain values, not curve heights.


B
Lower bound / 2^k (normalized)
1.5
1.4
1.3
1.2
1.1
1.0
23 exact
Gain
+3
+3
+3
+4
+5
+27
+30
+31
4664
Refined 9564
Refined 4647
35
67
131
267
543
1112
2275
4633
9539
32
64
128
263
538
1085
2245
4
5
6
7
8
9
10
11
12
13
Property B: hyperedge size k
Common reference envelope L_{ref}
Our refined criterion: certified evaluation
Small k (paper/certificates)


A
Lower bound on m(k)
2^{70}
2^{60}
2^{50}
2^{40}
2^{30}
2^{20}
2^{10}
2^{0}
Almost coincident
see main plot
m(4)=23
exact
Five-bound envelope: $B_{RS}+1$
for every k≥12
Uniform theorem k≥12
4
12
17
24
32
40
48
56
64
Property B: hyperedge size k
Known exact value
Small k (paper/certificates)
Uniform theorem k≥12 (Lean)
Common reference envelope
Certified RS reference bound
Our refined criterion:
certified evaluation k=12,13